



FUDAN
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Chiral symmetry breaking in accelerating and rotating frames

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Based on:

PRD 113(2026) 034005 Zhu, Z. B., Chen, H. L.,
& Huang, X. G.

PRD 114 (2026) 054012 Chen, H. L.,
Fukushima, K., Gao, Y. H., Huang, X. G.,
Shimada, Y., & Zhu, Z. B.

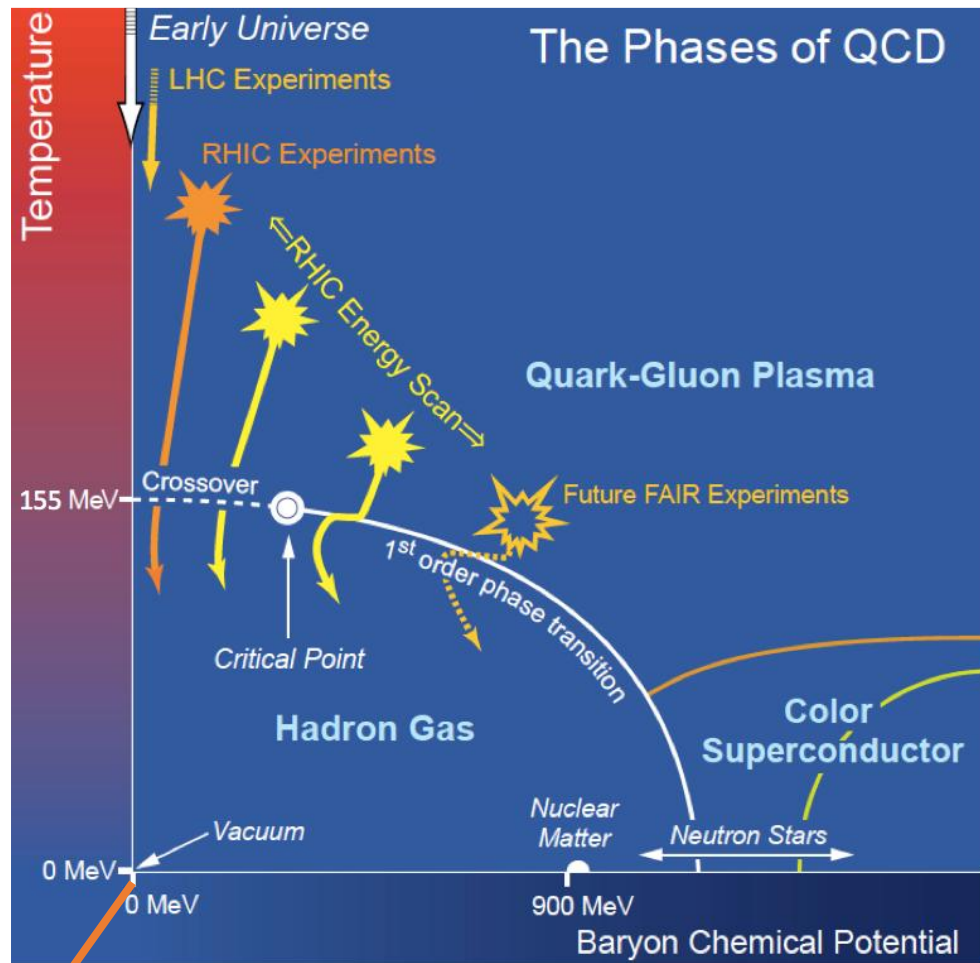


Outline



- QCD phase transition
- Uniform acceleration, Unruh effect, and Rindler coordinates
- Chiral symmetry under acceleration
 - $NL\sigma$ model analysis and NJL model analysis**
 - The dependence of Regularization scheme**
- Under rotation and acceleration
- Gluon under acceleration
- Summary

QCD phase transition



(Ω, a, B, E)

Two important non-perturbative properties in QCD : **color confinement** and **chiral symmetry breaking**

Order parameter for **chiral phase transition**:

Chiral condensate $\langle \bar{\psi}\psi \rangle$

QGP phase $\langle \bar{\psi}\psi \rangle = 0$ Chiral symmetry **restored**

Hadronic phase $\langle \bar{\psi}\psi \rangle \neq 0$ Chiral symmetry **breaking**

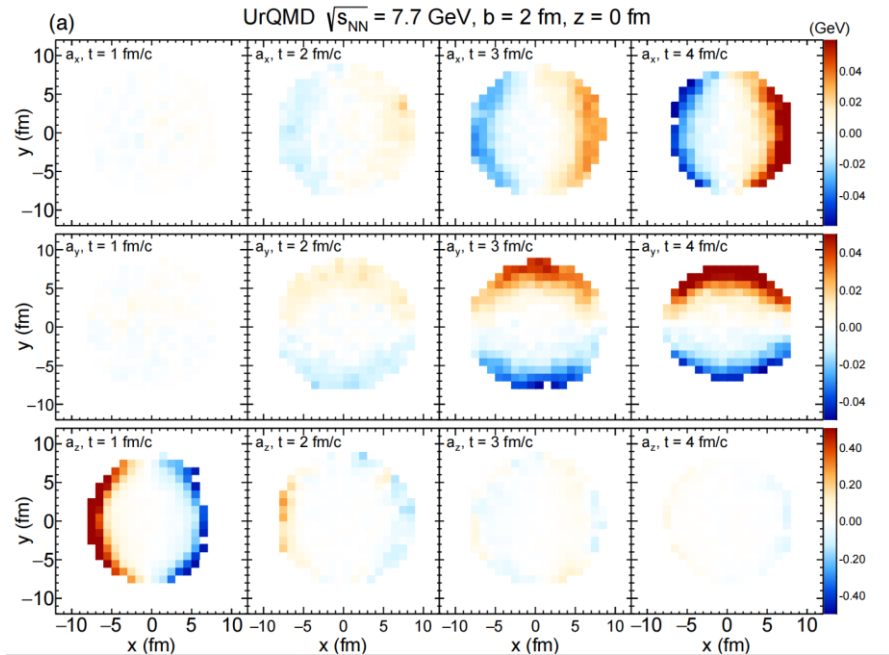
Order parameter for **deconfinement phase transition**:

Polyakov loop $L(\vec{x}) = \frac{1}{N_c} \text{tr} \mathcal{P}(\vec{x})$ with $\mathcal{P}(\vec{x}) = \text{P} e^{ig \int_0^\beta dt A_0(t, \vec{x})}$

Confinement: $\langle L \rangle = 0$ center symmetric

Deconfinement: $\langle L \rangle \neq 0$ center symmetry **breaking**

Rotation and Acceleration in Heavy-Ion Collisions

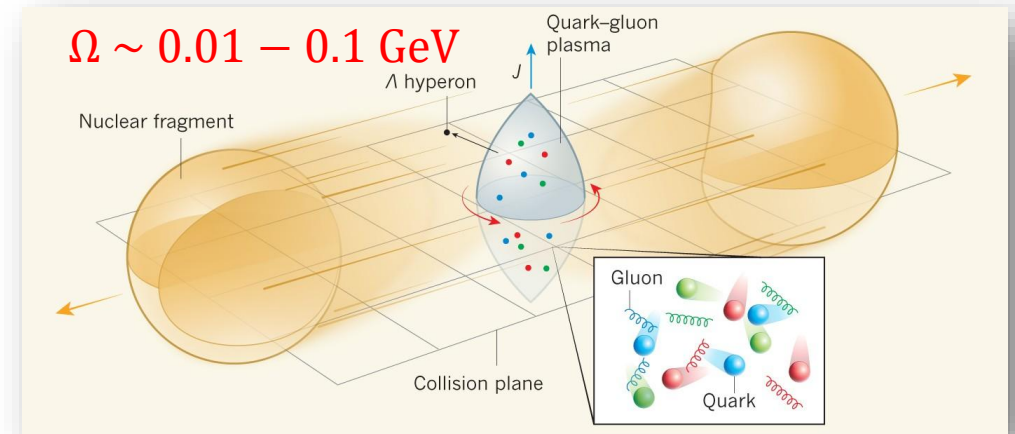


The UrQMD simulation result for the acceleration distribution.

Taken from arXiv:2604.00302

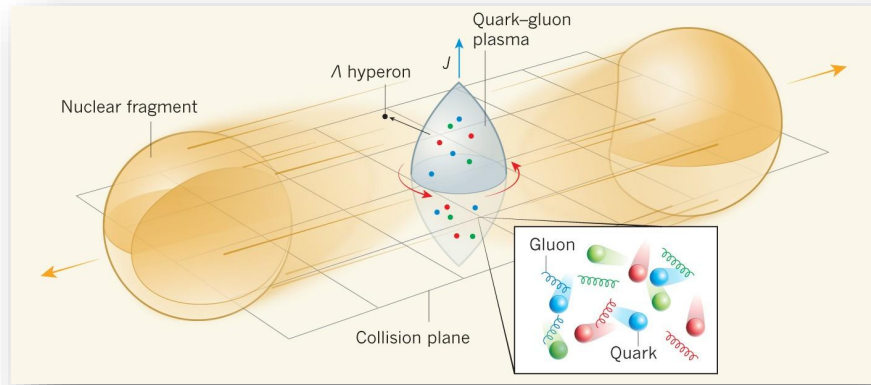
In heavy ion collisions, the acceleration can reach a scale of about **0.1~0.4 GeV**.

If chromoelectric fields are taken into account, a can even reach 1GeV



In a non-central collision, there exists a rapid rotation.

Acceleration and Rotation Effects



$$H \rightarrow H - J \cdot \Omega$$

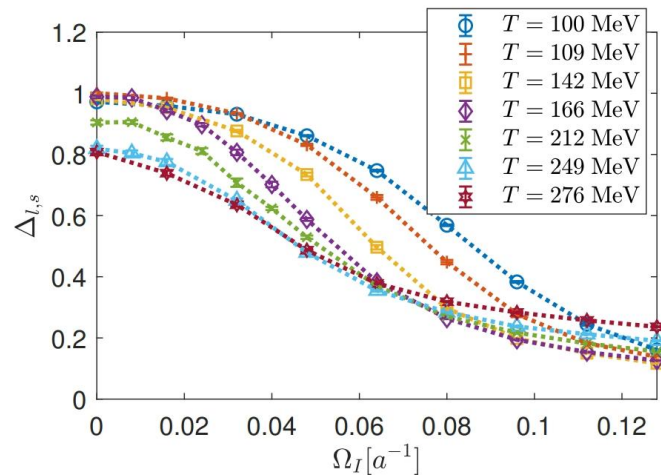
• In a rotating system, the Hamiltonian acquires an additional term, which allows rotation to be interpreted as an effective chemical potential. Therefore, it is natural to expect that rotation will have a nontrivial impact on the QCD phase transition.



$$T = a/2\pi$$

• According to the Unruh effect, a uniformly accelerated observer with proper acceleration a perceives the Minkowski vacuum as a thermal bath at temperature $T = a/2\pi$. This suggests that acceleration may also influence the phase transition.

Puzzle of rotation



Ji-Chong Yang and Xu-Guang Huang arxiv:2307.05755

• **Effective models:** Rotation **suppresses** chiral condensate $\rightarrow T_c$ decreases with Ω

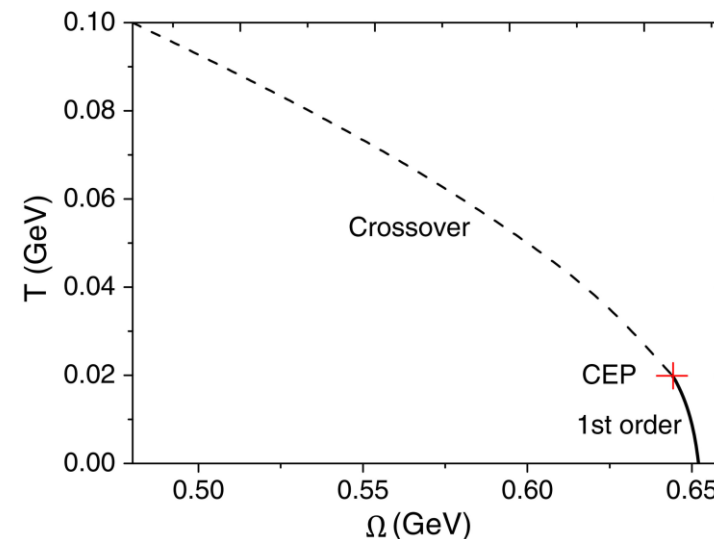
Lattice QCD (imaginary rotation) :

As Ω_I increases:

• Chiral condensate $\downarrow$, Polyakov loop $\uparrow \rightarrow$ Analytic continuation would imply real rotation **enhances** chiral symmetry breaking.

Lattice VS model

$\Omega \uparrow \rightarrow T_c \downarrow$ VS $\Omega \uparrow \rightarrow T_c \uparrow$

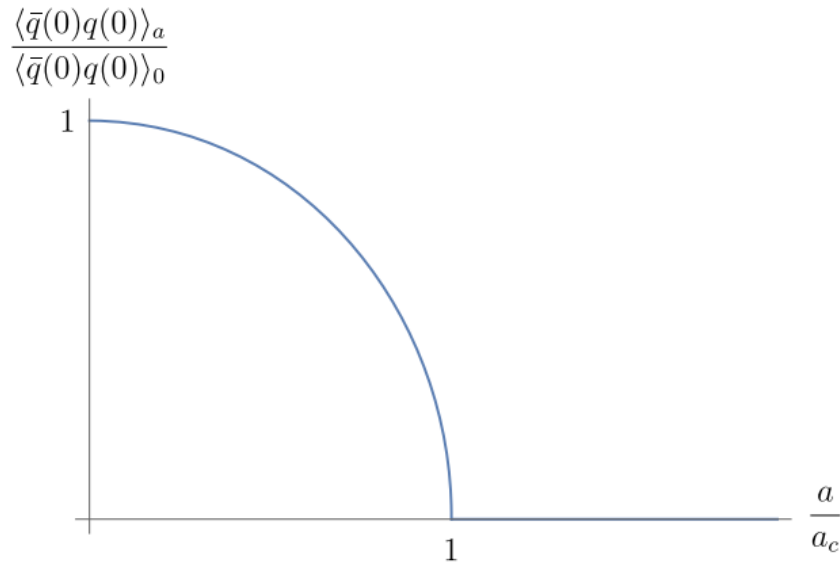


Y. Jiang and J. Liao, Phys. Rev. Lett. 117, 192302 (2016) arXiv:1606.03808

Possible sources of discrepancy:

- Limitations of mean-field approximation?
- Limitations of effective models?
- Can lattice results be extrapolated to real rotation?
- Role of gluons? ...

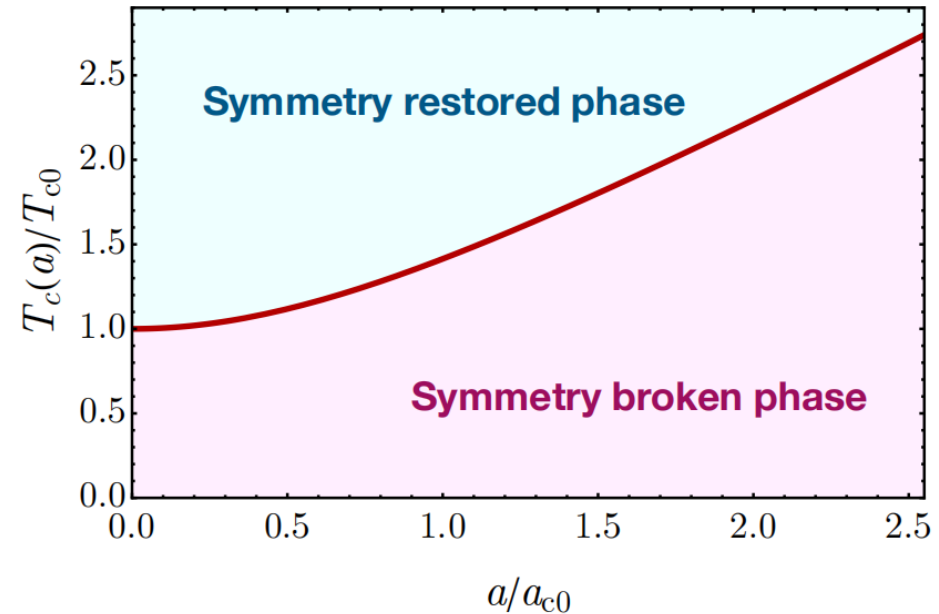
Puzzle of acceleration



Casado-Turrión, Adrián, and Antonio Dobado. *Physical Review D* 99.12 (2019): 125018.

Restoration vs. Breaking

$a \uparrow \rightarrow T_c \downarrow$ VS $a \uparrow \rightarrow T_c \uparrow$



Chernodub, Maxim N. *arXiv:2501.16129* (2025)

- Some works: $a \uparrow \rightarrow$ **restoration** of chiral symmetry
- Other works: $a \uparrow \rightarrow$ **breaking** of chiral symmetry

The renormalization scheme is the key issue.

Framework

We use the quantum field theory in curved spacetime and adopt two low-energy effective models.

We work in Rindler spacetime(accelerating coordinate).

$$ds^2 = \xi^2 dt^2 - d\xi^2 - dx_{\perp}^2$$

	NLσM (Bosonic)	NJL (Fermionic)
DOF	σ, π^a	$\psi, \bar{\psi}$
Order parameter	$\sigma = \langle \sigma \rangle$	$\sigma = -G_{\pi} \langle \bar{\psi} \psi \rangle$
Method	Klein-Gordon eq. in curved spacetime	Dirac eq. in curved spacetime
Why used	Cross-check, scalar case	Quark DOF, more realistic
NLσ model Lagrangian:	$\mathcal{L}_{NL\sigma M} = -\frac{1}{2} \Phi^T \square \Phi - M_{\pi}^2 f_{\pi} \sigma$	
NJL model Lagrangian:	$\mathcal{L}_{NJL} = \bar{\psi} \left[i \gamma^{\mu} \nabla_{\mu} - m_0 \right] \psi + \frac{G_{\pi}}{2} \left[(\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma^5 \tau \psi)^2 \right]$	

Route:
Solve KG/Dirac eq. → Propagator G(x,x) → Gap eq. → σ(a, T)

The propagator G(x,x) is divergent, which means we need to introduce the choice of a renormalization scheme:

Subtraction of Rindler vacuum vs Minkowski vacuum
→ two different phase diagrams

Uniformly accelerated motion

Equation of motion:

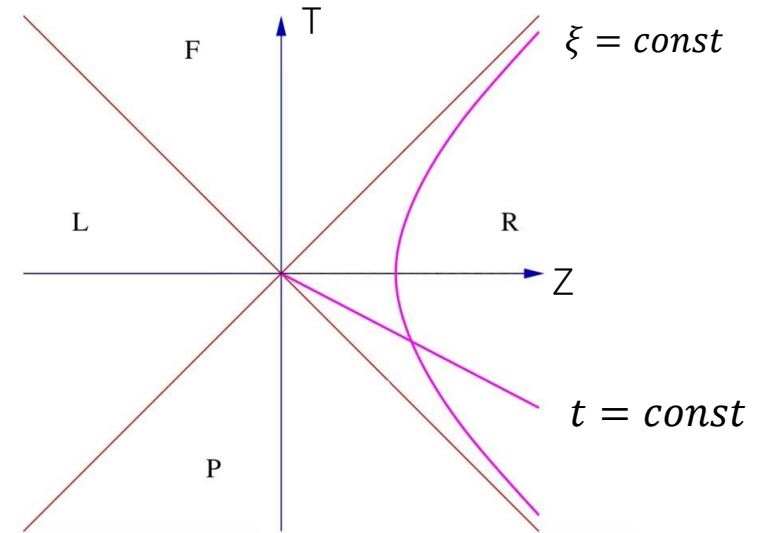
$$u^\mu = \frac{dx^\mu}{d\tau} \quad \alpha^\mu = \frac{du^\mu}{d\tau} \quad \alpha^\mu \alpha_\mu = -a^2$$

For motion along the (z) direction, the worldline is

$$T(\tau) = \frac{1}{a} \sinh(a\tau), Z(\tau) = \frac{1}{a} \cosh(a\tau).$$

The trajectory is a hyperbola in Minkowski spacetime:

$$Z^2 - T^2 = \frac{1}{a^2}$$



Rindler coordinates

A family of uniformly accelerated observers defines the Rindler coordinate system:

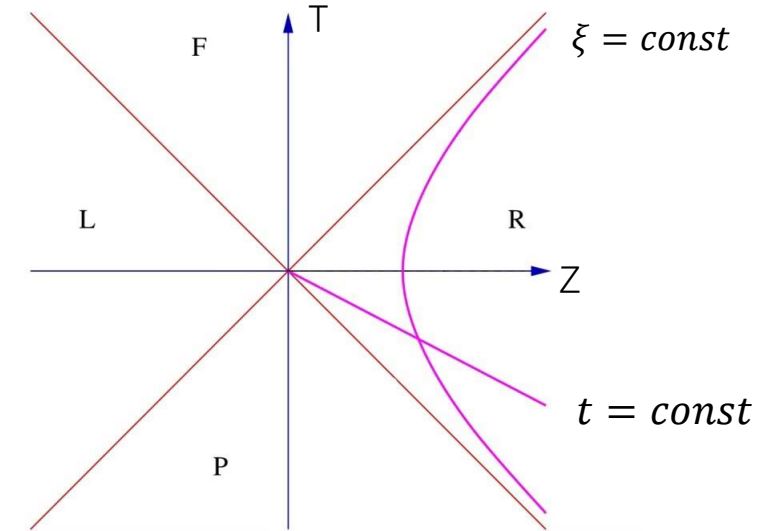
$$T = \xi \sinh t,$$

$$Z = \xi \cosh t.$$

The line element becomes $ds^2 = \xi^2 dt^2 - d\xi^2 - d\mathbf{x}_\perp^2$

An observer at fixed ξ follows a hyperbolic worldline with

$$\xi = 1/a \quad t = a\tau$$



Vacuum Ambiguity

The **Rindler vacuum** $|0\rangle_R$ is not equivalent to the **Minkowski vacuum** $|0\rangle_M$.

The particle number operator satisfies:

Define the **creation** and **annihilation** operators for Rindler spacetime: $a_R(\omega)$ 和 $a_R^\dagger(\omega)$,

According to the definition, it satisfies: $a_R|0\rangle_R=0$

$${}_M\langle 0|a_R^\dagger a_R|0\rangle_M \sim \left(\exp\left(\frac{2\pi\omega}{a}\right) - 1\right)^{-1} \quad (\text{For boson})$$

Thermal States in Rindler Coordinates

After Wick rotation $t = -it_E$, the Euclidean metric is

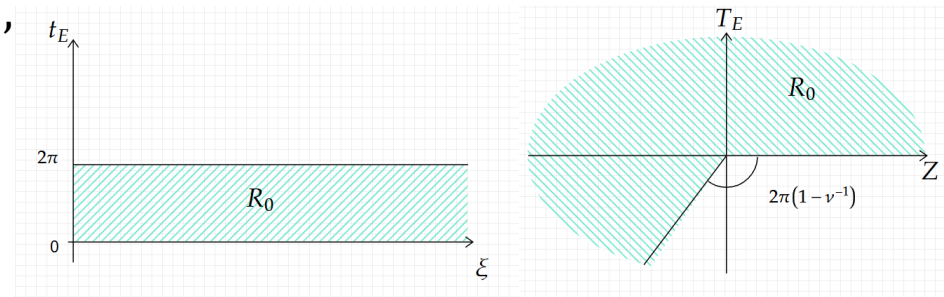
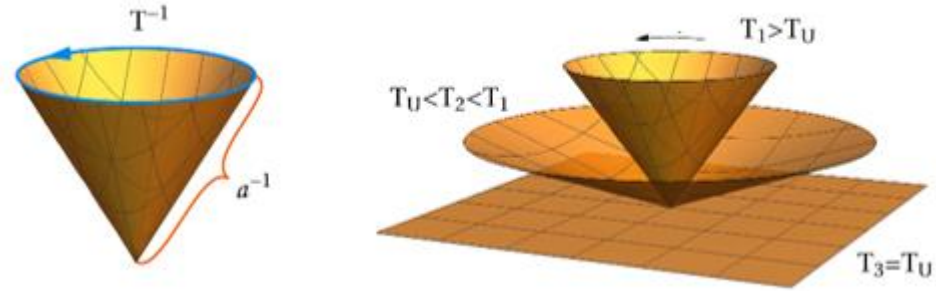
$$ds^2 = -\frac{\xi^2}{\nu^2} dt_E^2 - d\xi^2 - dx_1^2 - dx_2^2$$

where the Euclidean time is periodic, $T_E = \xi \sin t_E/\nu$,

$$0 < t_E < 2\pi, \quad \nu = 2\pi T/a \quad Z = \xi \cos t_E/\nu.$$

Regularity condition

- $\nu = 1 (T = T_U)$
 - the Euclidean geometry is smooth;
 - the conical singularity disappears.
- $\nu > 1 (T > T_U)$
 - a positive angular deficit (conical geometry) appears.
- $\nu < 1 (T < T_U)$
 - the angular deficit becomes negative;
 - The geometry meaning becomes unclear
 - Indicating an unstable stage



To avoid a negative angular deficit, we impose $\nu \geq 1$ in this work, which leads to the condition $T \geq T_U$.

NL σ model analysis

The NL σ model Lagrangian: $\mathcal{L}_{NL\sigma M} = -\frac{1}{2}\Phi^T \square \Phi - M_\pi^2 f_\pi \sigma \quad \Phi^T = (\pi^a, \sigma)$

The partition function: $Z = \int \mathcal{D}[\Phi] \delta[\Phi^T \Phi - f_\pi^2] \exp \left(i \int d^4x \mathcal{L}_{NL\sigma M} \right)$

introducing an auxiliary field $\lambda(x)$ and integrating out the pion fields and taking the large- N limit

The effective action: $\Gamma[\sigma, \lambda] = \int d^4x \left(-\frac{1}{2}\sigma \square \sigma + \frac{\lambda}{2}(\sigma^2 - f_\pi^2) + \frac{N}{2} \ln \frac{-\square + \lambda}{-\square} - M_\pi^2 f_\pi \sigma \right)$

The gap equation:

$$\frac{\delta \Gamma}{\delta \sigma} = -\square \sigma + \lambda \sigma - M_\pi^2 f_\pi = 0,$$

$$\mathbf{M}_\pi = \mathbf{0}, \square \sigma = \mathbf{0} \rightarrow \lambda = \mathbf{0}$$

We assume $\square \sigma \approx \mathbf{0}$ in acceleration case

$$\frac{\delta \Gamma}{\delta \lambda} = \frac{\sigma^2 - f_\pi^2}{2} + \frac{N}{2} G(x, x; \lambda) = 0$$

$$\sigma^2 = f_\pi^2 - N G(x, x; 0)$$

NL σ model analysis

$$\square\phi \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = \frac{1}{\sqrt{-g}}\partial_\mu [g^{\mu\nu}\sqrt{-g}\partial_\nu\phi]$$

$$ds^2 = \xi^2 dt^2 - d\xi^2 - dy^2 - dz^2$$

The explicit form of the KG equation in Rindler coordinate is

$$\left(-\frac{1}{\xi^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial \xi^2} + \frac{1}{\xi}\frac{\partial}{\partial \xi} + \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} - m^2\right)\phi = 0$$

The eigenfunctions

$$\phi = \frac{1}{\sqrt{2\omega}} \frac{1}{2\pi} \sqrt{\frac{2\omega \sinh \pi\omega}{\pi^2}} K_{i\omega}(m_\perp \xi) e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}$$

K_μ is the modified Bessel function satisfying the equation:

$$z^2 \frac{d^2 K_\nu(z)}{dz^2} + z \frac{dK_\nu(z)}{dz} = (z^2 + \nu^2) K_\nu(z)$$

The Green function:

$$G_E(x_E, x'_E) = \int_0^\infty d\omega \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{\sinh \pi\omega}{\pi^2} K_{i\omega}(m_\perp \xi) K_{i\omega}(m_\perp \xi') e^{-\omega|t_E - t'_E| + i\mathbf{k}\cdot(\mathbf{x} - \mathbf{x}')}$$

NL σ model analysis

The Green function:

$$\begin{aligned} G_\nu &= \sum_n G_E(t_E - t'_E + \beta_R n) \\ &= \int_0^\infty d\omega \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{\cosh \omega(|t_E - t'_E| - \beta_R/2)}{\sinh(\beta_R \omega/2)} \frac{\sinh \pi \omega}{\pi^2} K_{i\omega}(m_\perp \xi) K_{i\omega}(m_\perp \xi') e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{x}')}. \end{aligned}$$

The diagonal part of Green function:

$$G_\nu(x_E, x_E) = \int_0^\infty \frac{d\omega}{\pi^2} \frac{d^2\mathbf{k}}{(2\pi)^2} \coth(\omega \beta_R/2) \sinh(\pi \omega) K_{i\omega}^2(m_\perp \xi).$$



Massless limit

$$G_\nu(x_E, x_E; m = 0) = \int_0^\infty \frac{d\omega}{\pi^2} \frac{1}{(2\pi)} \coth(\omega \beta_R/2) \frac{\pi \omega}{2\xi^2}$$

Renormalization in the NL σ Model

The diagonal part of Green function: $G_\nu(x_E, x_E; m=0) = \int_0^\infty \frac{d\omega}{\pi^2} \frac{1}{(2\pi)} \coth(\omega\beta_R/2) \frac{\pi\omega}{2\xi^2}$

I: Subtract the Rindler vacuum expectation value

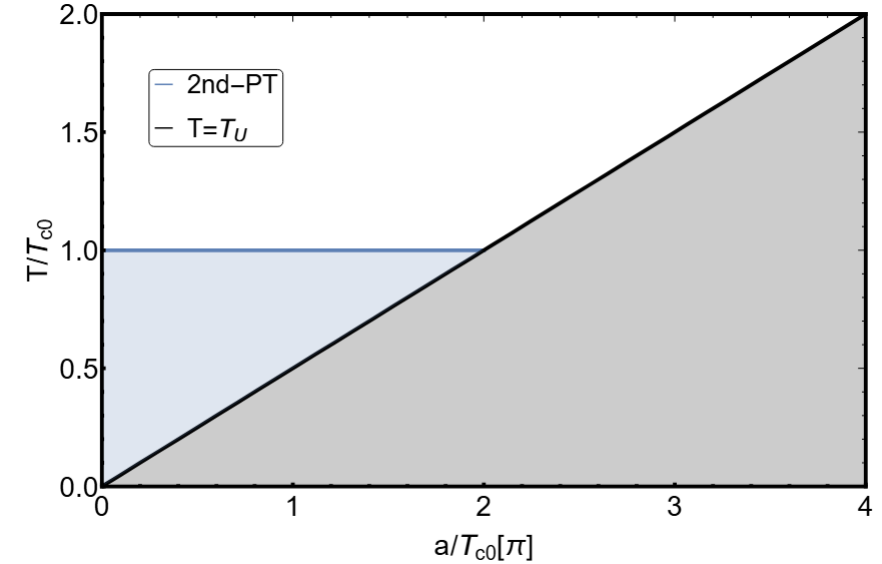
$$\langle \hat{O} \rangle_{ren} = \langle \hat{O} \rangle_\beta - \langle \hat{O} \rangle_R$$

The position of the accelerating observer:

$$\sigma^2 = f_\pi^2 - \frac{T^2}{a^2 \xi^2} \frac{N}{12}$$

$$\xi = 1/a$$

$$T_c = a\xi \sqrt{\frac{12f_\pi^2}{N}}$$



II: Subtract the Minkowski vacuum expectation value

$$\langle \hat{O} \rangle_{ren} = \langle \hat{O} \rangle_\beta - \langle \hat{O} \rangle_M$$

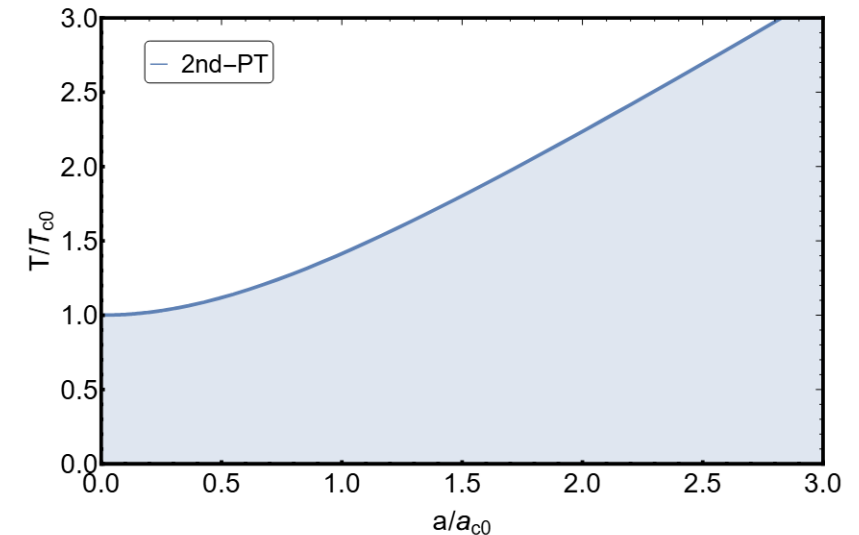
$$\sigma^2 = f_\pi^2 - \frac{1}{(a\xi)^2} \frac{N}{12} \left(T^2 - \frac{a^2}{(2\pi)^2} \right)$$

The position of the accelerating observer:

$$T_c(a) = \sqrt{\frac{12f_\pi^2}{N} + \left(\frac{a}{2\pi}\right)^2}$$

$$= \sqrt{T_{c0}^2 + \left(\frac{a}{2\pi}\right)^2}$$

$$\xi = 1/a$$



Different vacuum choices lead to different result.

NJL model analysis

The Green's function G is

$$G_\nu = \int_0^\infty d\omega \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{\sinh(\beta_R\omega/2 - \omega|t_E - t'_E|)}{\cosh(\beta_R\omega/2)} \frac{\sinh \pi(\omega + i\gamma^0\gamma^3/2)}{\pi^2} K_{i\omega - \gamma^0\gamma^3/2}(m_\perp\xi) K_{i\omega - \gamma^0\gamma^3/2}(m_\perp\xi') e^{i\mathbf{k}\cdot(\mathbf{x}_1 - \mathbf{x}_2)}$$

I: Subtract the Rindler vacuum expectation value

$$\begin{aligned} G_\nu(x_E, x_E; m=0) &= \int_0^\infty d\omega \frac{\omega}{\pi^2 \xi^2} \tanh(\beta_R\omega/2) \\ &= \int_0^\Lambda d\omega \frac{\omega}{\pi^2 a^2 \xi^2} + \int_0^\infty d\omega \frac{\omega}{\pi^2 a^2 \xi^2} \frac{2}{e^{\beta\omega} + 1} \\ &= \frac{1}{6a^2 \xi^2} \left(\frac{3\Lambda^2}{\pi^2} - T^2 \right) \end{aligned}$$

The gap equation: $\frac{1}{G_\pi} = \frac{1}{6a^2 \xi^2} \left(\frac{3\Lambda^2}{\pi^2} - T^2 \right)$

The critical temperature is $T_c = a\xi \sqrt{\frac{3\Lambda^2}{\pi^2} - \frac{6}{G}}$

II: Subtract the Minkowski vacuum expectation value

$$G_\nu^{ren}(x_E, x_E; m=0) = \frac{1}{6a^2 \xi^2} \left(\frac{3\Lambda^2}{\pi^2} - T^2 + \frac{a^2}{(2\pi)^2} \right)$$

The critical temperature is $T_c(a) = \sqrt{\frac{3\Lambda^2}{\pi^2} - \frac{6}{G} + \frac{a^2}{(2\pi)^2}}$
 $= \sqrt{T_{c0}^2 + \left(\frac{a}{2\pi}\right)^2},$

Renormalization scheme

I: Subtract the Rindler vacuum expectation value

$$\langle \hat{O} \rangle_{ren} = \langle \hat{O} \rangle_{\beta} - \langle \hat{O} \rangle_R \quad T_c = a\xi \sqrt{\frac{3\Lambda^2}{\pi^2} - \frac{6}{G_{\pi}}}$$

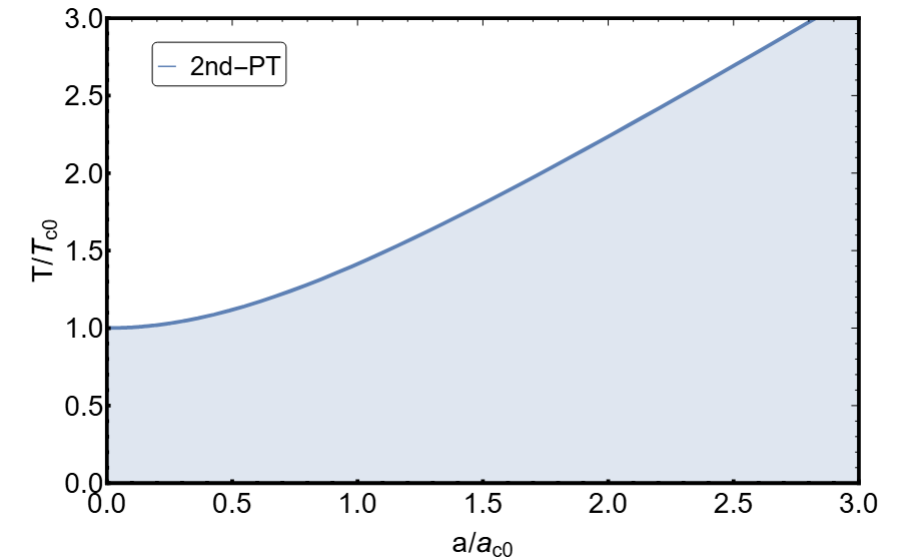
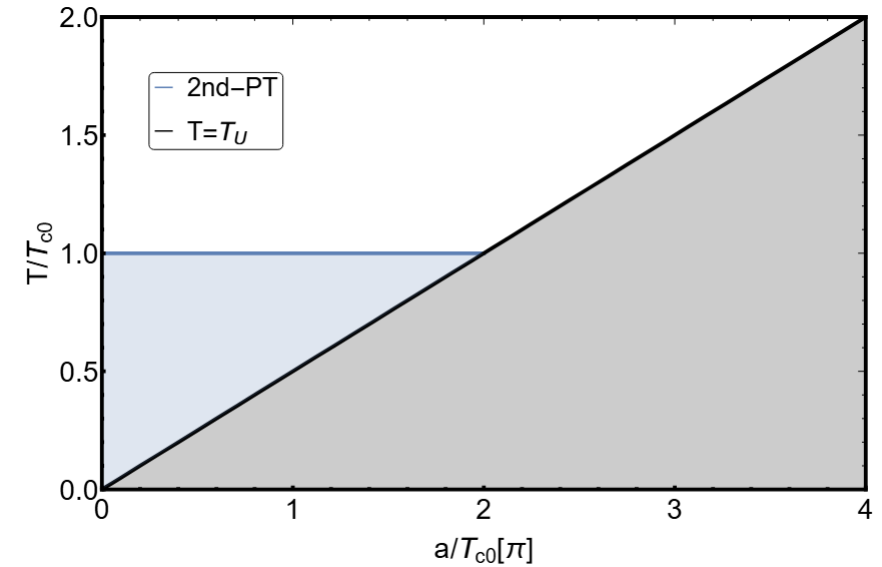
T_c is independent of acceleration

II: Subtract the Minkowski vacuum expectation value

$$\langle \hat{O} \rangle_{ren} = \langle \hat{O} \rangle_{\beta} - \langle \hat{O} \rangle_M \quad T_c(a) = \sqrt{\frac{3\Lambda^2}{\pi^2} - \frac{6}{G} + \frac{a^2}{(2\pi)^2}}$$

$$T_c \text{ is enhanced by acceleration} \quad = \sqrt{T_{c0}^2 + \left(\frac{a}{2\pi}\right)^2}$$

The conclusion is independent of the effective model.



NJL Model under Acceleration and Rotation

Framework

Metric:
$$g_{\mu\nu} = \begin{pmatrix} \xi^2 - r^2 \bar{\Omega}^2 & \bar{\Omega} y & -\bar{\Omega} x & 0 \\ \bar{\Omega} y & -1 & 0 & 0 \\ -\bar{\Omega} x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



Solving the Dirac equation



Gap equation



Numerical solution

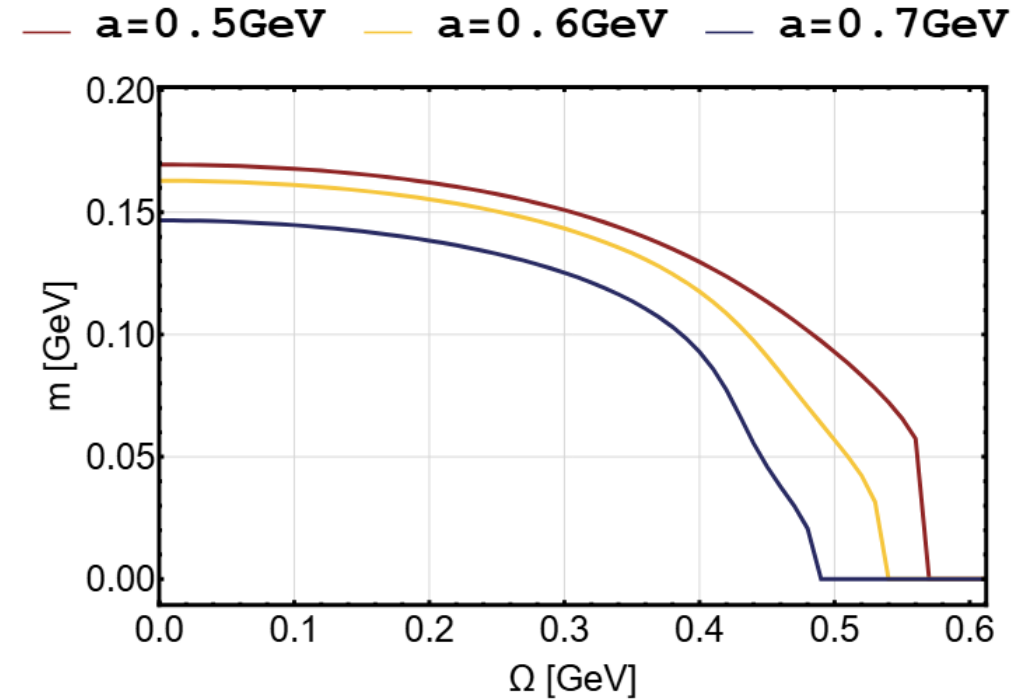
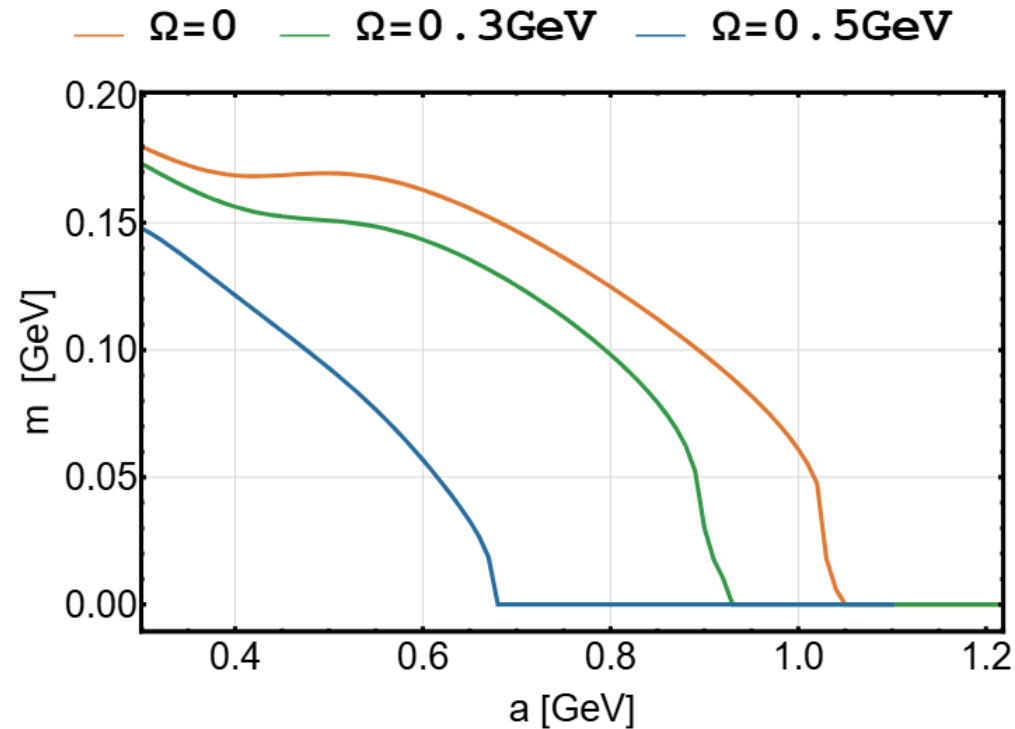
Assumptions

Acceleration and rotation are parallel.

We subtract the Rindler vacuum expectation

$$T = T_U.$$

Result



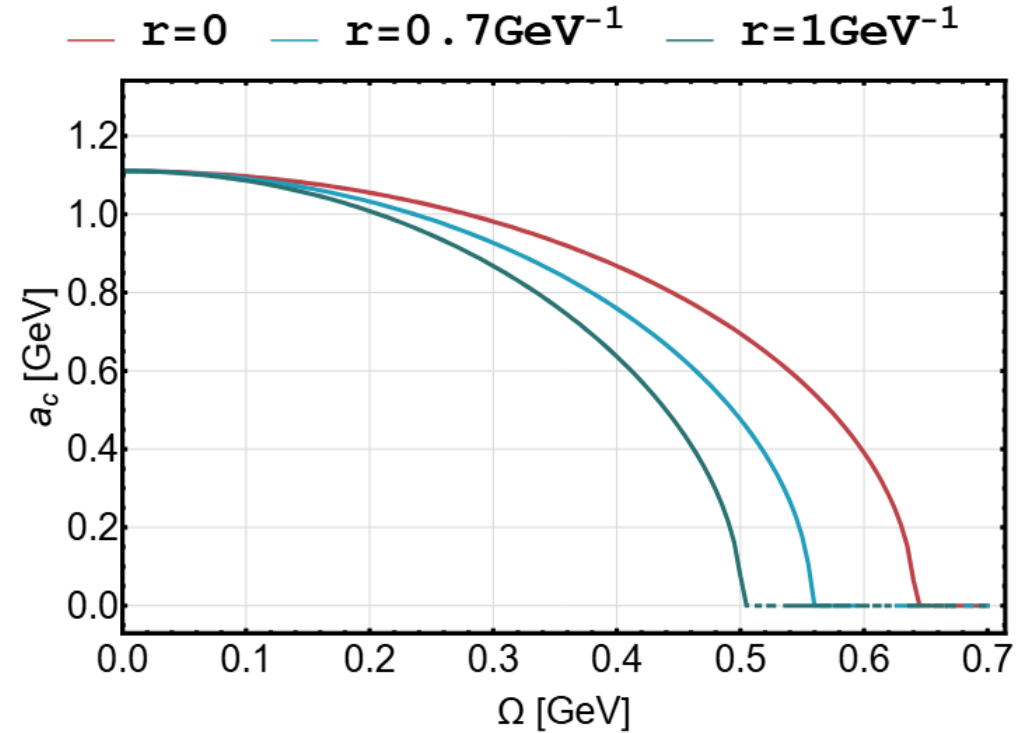
- Acceleration induces thermalization
- Rotation induces an effective chemical potential
- Both acceleration and rotation restore chiral symmetry

Analytical Result in the Massless Limit

The gap equation can be solved analytically.

$$G_{\pi} \left(\frac{\Lambda^2}{2\pi^2} - \frac{a^2 (r^4 \Omega^4 + 1) - r^2 \Omega^4 + 3\Omega^2}{24\pi^2 (r^2 \Omega^2 - 1)^2} \right) = 1.$$

The critical acceleration is obtained directly from this equation.



Rotation lowers the critical acceleration.

The effects of rotation become increasingly significant with increasing radius.

Summary I

- ✓ We developed a unified framework to study chiral symmetry under acceleration and rotation.
- ✓ Different renormalization schemes lead to different phase diagrams.
- ✓ Under the Rindler vacuum subtraction, both acceleration and rotation favor chiral symmetry restoration.

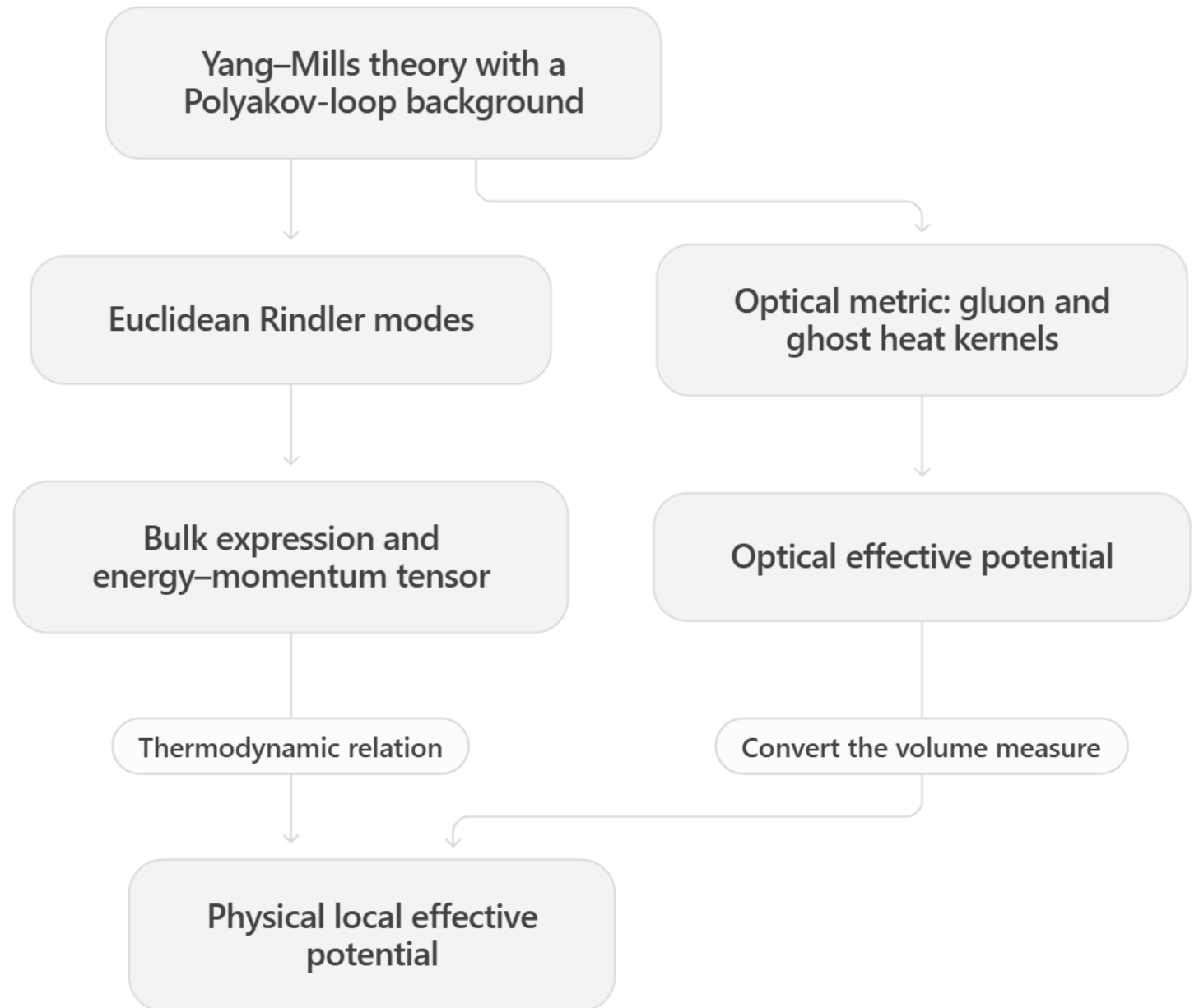
Framework

Subtle:

$$\ln Z_{\text{opt}} \neq \ln Z_{\text{ER}}$$

Relate to V_{eff}

Relate to T_τ^τ



Formalism I

Metric:

$$ds_E^2 = dx^2 + dy^2 + dz^2 + (az)^2 d\tau^2$$

One loop action:

$$\Gamma^{(1)} = \frac{1}{2} \text{Tr} \ln \Delta_{\text{gluon}} - \text{Tr} \ln \Delta_{\text{ghost}}$$

We expand the gauge field:

$$A_\mu = \bar{A}_\mu + Q_\mu, \quad \bar{A}_i = 0, \quad \bar{A}_\tau = \frac{\phi \cdot \mathbf{H}}{g\beta_0} \quad \leftarrow \text{Cartan subalgebra}$$

$$\partial_\tau \longrightarrow D_{B\tau} = \partial_\tau + i \frac{\phi \cdot H}{\beta} \longrightarrow G_{B\tau} = \nabla_\tau + i \frac{\phi \cdot H}{\beta}$$

$$D_\tau Q_\mu^\alpha = \left(\partial_\tau + i \frac{\phi \cdot \alpha}{\beta_0} \right) Q_\mu^\alpha$$

$$\omega_n^\alpha = \frac{2\pi n + \phi \cdot \alpha}{\beta_0} \quad \leftarrow \text{The root of Cartan subalgebra}$$

The Cartan background shifts the Matsubara frequencies of the gluon root modes.

Formalism II

$$\mathcal{D}_s = -\frac{1}{a^2 z^2} D_\tau^2 - \partial_x^2 - \partial_y^2 - \partial_z^2 - \frac{1}{z} \partial_z$$

$$\ln Z_{\text{ER}} = \underbrace{\text{Tr} \ln \mathcal{D}_s}_{\text{ghost}} - \frac{1}{2} \underbrace{\text{Tr} \ln \mathcal{D}_v}_{\text{gluon}}$$

$$\mathcal{D}_v = \begin{pmatrix} \mathcal{D}_s & 0 & 0 & 0 \\ 0 & \mathcal{D}_s & 0 & 0 \\ 0 & 0 & \mathcal{D}_s + \frac{1}{z^2} & \frac{2}{z} D_\tau \\ 0 & 0 & -\frac{2}{a^2 z^3} D_\tau & \mathcal{D}_s - \frac{2}{z} \partial_z \end{pmatrix}$$

Conformal transformation

$$ds_{\text{Rindler}}^2 = -(az)^2 dt^2 + dx^2 + dy^2 + dz^2 \quad \longrightarrow \quad \begin{aligned} ds_{\text{opt}}^2 &= (g_{\text{opt}})_{\mu\nu} dx^\mu dx^\nu = d\tau^2 + h_{ij} dx^i dx^j, \\ h_{ij}(x) &= \frac{1}{(az)^2} \delta_{ij}. \end{aligned}$$

Subtle: $\ln Z_{\text{opt}} \neq \ln Z_{\text{ER}}$

Cartan subalgebra

$$L(\phi) = \frac{1}{N_c} \text{tr} \exp \left(ig \int_0^{\beta_0} \bar{A}_\tau d\tau \right) = \frac{1}{N_c} \text{tr} e^{i\phi \cdot \mathbf{H}}$$

For SU(2):

$$H = T^3 = \frac{\sigma^3}{2} = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad g\beta_0 \bar{A}_\tau = \phi H$$

$$\alpha = \pm 1$$

$$L_{SU(2)} = \cos \frac{\phi}{2}$$

For SU(3):

$$H_3 = \frac{\lambda^3}{2} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$H_8 = \frac{\lambda^8}{2} = \frac{1}{2\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$\alpha = \pm(1, 0), \quad \pm \left(\frac{1}{2}, \frac{\sqrt{3}}{2} \right), \quad \pm \left(-\frac{1}{2}, \frac{\sqrt{3}}{2} \right)$$

$$g\beta_0 \bar{A}_\tau = \phi_3 H_3 + \phi_8 H_8 = \text{diag} \left(\frac{\phi_3}{2} + \frac{\phi_8}{2\sqrt{3}}, -\frac{\phi_3}{2} + \frac{\phi_8}{2\sqrt{3}}, -\frac{\phi_8}{\sqrt{3}} \right) \quad \left| \quad L_{SU(3)} = \frac{1}{3} \left[2e^{i\phi_8/(2\sqrt{3})} \cos \frac{\phi_3}{2} + e^{-i\phi_8/\sqrt{3}} \right] \right.$$

Subtle between two method

ER calculation

Subtle: $\ln Z_{\text{opt}} \neq \ln Z_{\text{ER}}$ Optical calculation

$$ds_{\text{Rindler}}^2 = -(az)^2 dt^2 + dx^2 + dy^2 + dz^2.$$

$$ds_{\text{opt}}^2 = (g_{\text{opt}})_{\mu\nu} dx^\mu dx^\nu = d\tau^2 + h_{ij} dx^i dx^j,$$

$$h_{ij}(x) = \frac{1}{(az)^2} \delta_{ij}.$$

$$\begin{aligned} V_{\text{ER}} &= T_{\text{loc}}^4 \sum_{\alpha} \left[\frac{2\pi^2}{3} B_4 \left(\frac{\phi \cdot \alpha}{2\pi} \right) - \frac{\tilde{a}^2}{3} B_2 \left(\frac{\phi \cdot \alpha}{2\pi} \right) + \frac{11\tilde{a}^4}{720\pi^2} \right] \\ &= T_{\text{loc}}^4 \sum_{\alpha} \left[-\frac{\pi^2}{45} - \frac{\tilde{a}^2}{18} + \frac{(\phi \cdot \alpha)^4}{24\pi^2} - \frac{|\phi \cdot \alpha|^3}{6\pi} \right. \\ &\quad \left. + \left(\frac{1}{6} - \frac{\tilde{a}^2}{12\pi^2} \right) (\phi \cdot \alpha)^2 + \frac{\tilde{a}^2}{6\pi} |\phi \cdot \alpha| + \frac{11\tilde{a}^4}{720\pi^2} \right]. \end{aligned}$$

$$\begin{aligned} V_{\text{opt}} &= T_0^4 \sum_{\alpha} \left[\frac{2\pi^2}{3} B_4 \left(\frac{\phi \cdot \alpha}{2\pi} \right) - \tilde{a}^2 B_2 \left(\frac{\phi \cdot \alpha}{2\pi} \right) - \frac{11\tilde{a}^4}{240\pi^2} \right] \\ &= T_0^4 \sum_{\alpha} \left[-\frac{\pi^2}{45} - \frac{\tilde{a}^2}{6} + \frac{(\phi \cdot \alpha)^4}{24\pi^2} - \frac{|\phi \cdot \alpha|^3}{6\pi} \right. \\ &\quad \left. + \left(\frac{1}{6} - \frac{\tilde{a}^2}{4\pi^2} \right) (\phi \cdot \alpha)^2 + \frac{\tilde{a}^2}{2\pi} |\phi \cdot \alpha| - \frac{11\tilde{a}^4}{240\pi^2} \right]. \quad (39) \end{aligned}$$

The coincident relation:

$$V_{\text{eff}}(x) = (az)^{-4} V_{\text{opt}} = -\frac{3}{\beta_0} \int^{\beta_0} d\beta V_{\text{ER}}(x).$$

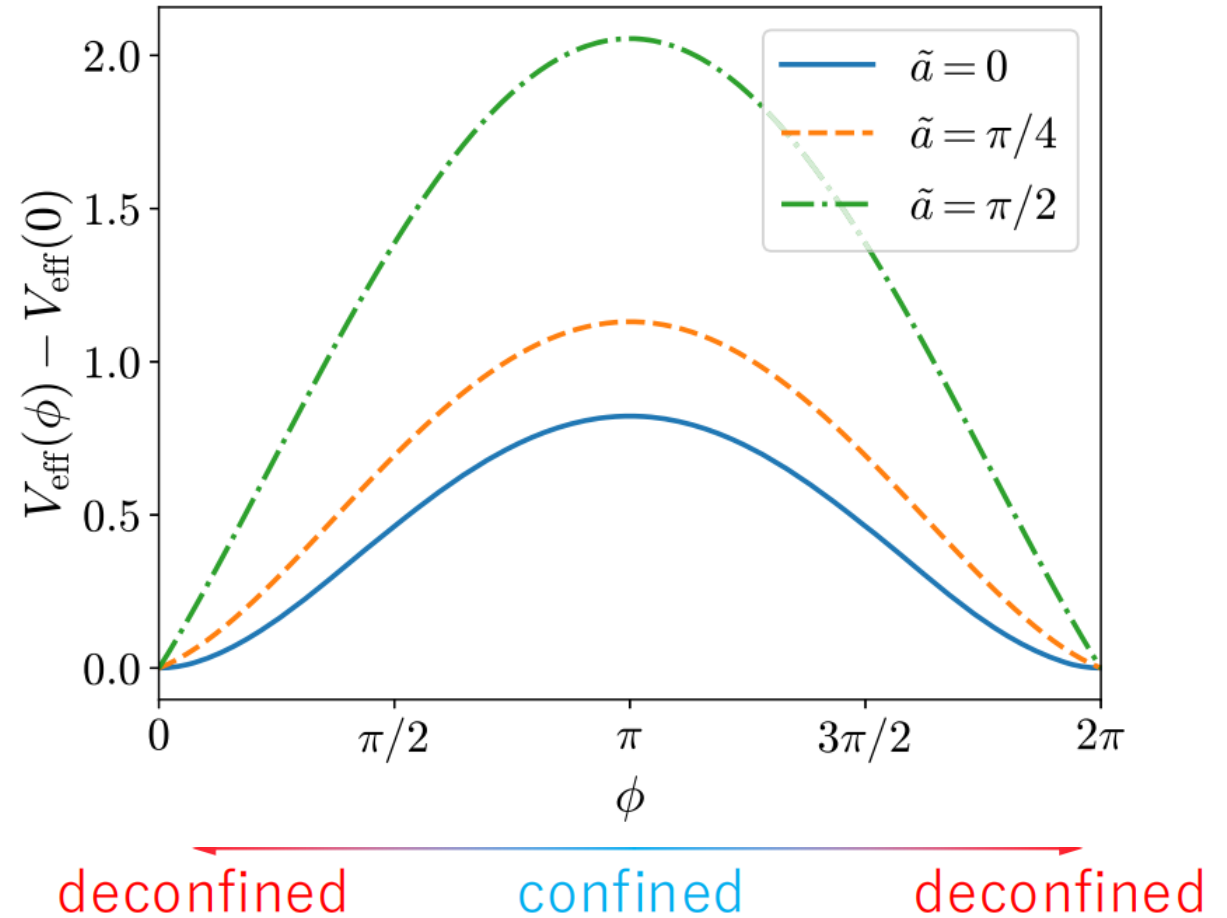
The ϕ is obtained by minimizing the V_{eff}

Real acceleration

$$V_{\text{eff}}(x) = T_{\text{loc}}^4 \sum_{\alpha} \left[\frac{2\pi^2}{3} B_4 \left(\frac{\phi \cdot \alpha}{2\pi} \right) - \tilde{a}^2 B_2 \left(\frac{\phi \cdot \alpha}{2\pi} \right) - \frac{11\tilde{a}^4}{240\pi^2} \right]$$

The system is always deconfined.

Acceleration enhances the deconfinement



Imaginary acceleration

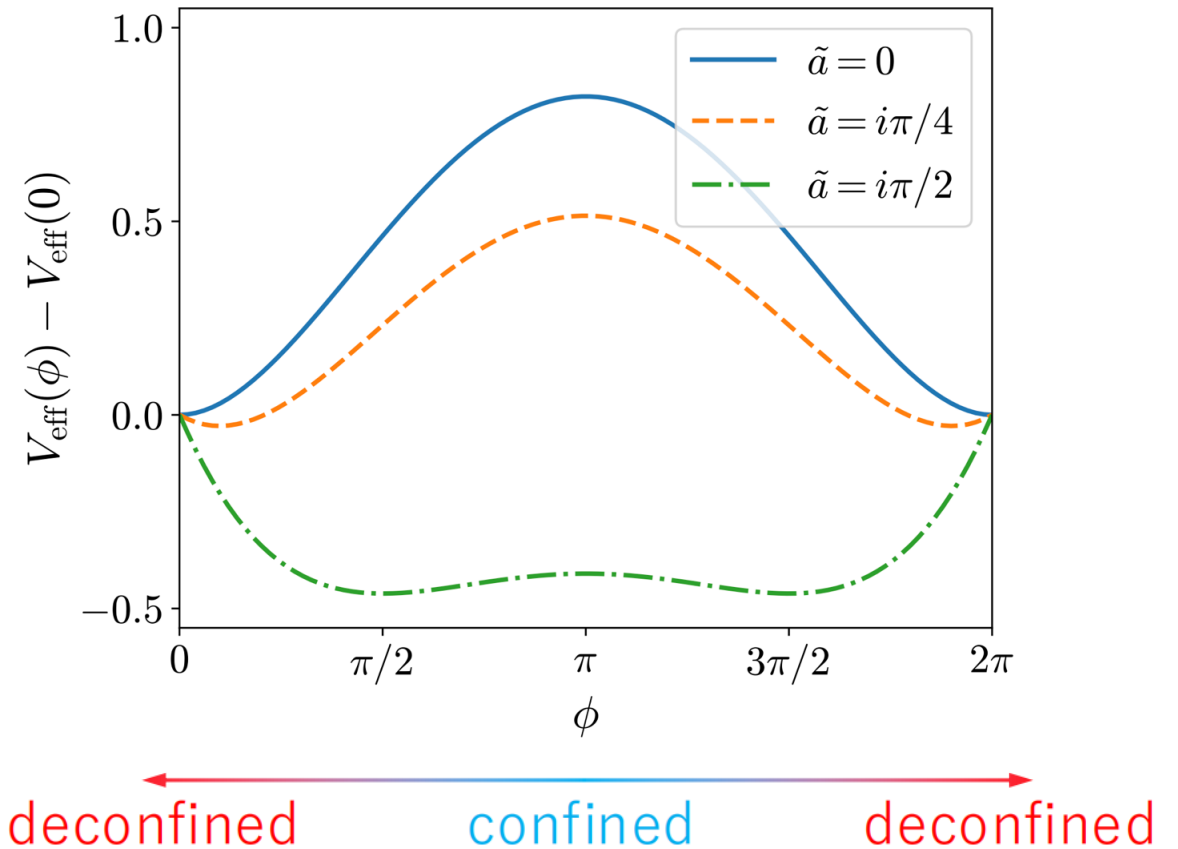
$$V_{\text{eff}}(x) = T_{\text{loc}}^4 \sum_{\alpha} \left[\frac{2\pi^2}{3} B_4\left(\frac{\phi \cdot \alpha}{2\pi}\right) - \tilde{a}^2 B_2\left(\frac{\phi \cdot \alpha}{2\pi}\right) - \frac{11\tilde{a}^4}{240\pi^2} \right]$$

$$\tilde{a} \rightarrow -i\tilde{a}_I$$

Sign change

$$\tilde{V}_{\text{eff}}(x) = 2T_{\text{loc}}^4 \left[\frac{2\pi^2}{3} B_4\left(\frac{\phi}{2\pi}\right) + \tilde{a}_I^2 B_2\left(\frac{\phi}{2\pi}\right) + \dots \right]$$

Imaginary acceleration will induce confinement.



Polyakov-loop under Imaginary acceleration and rotation

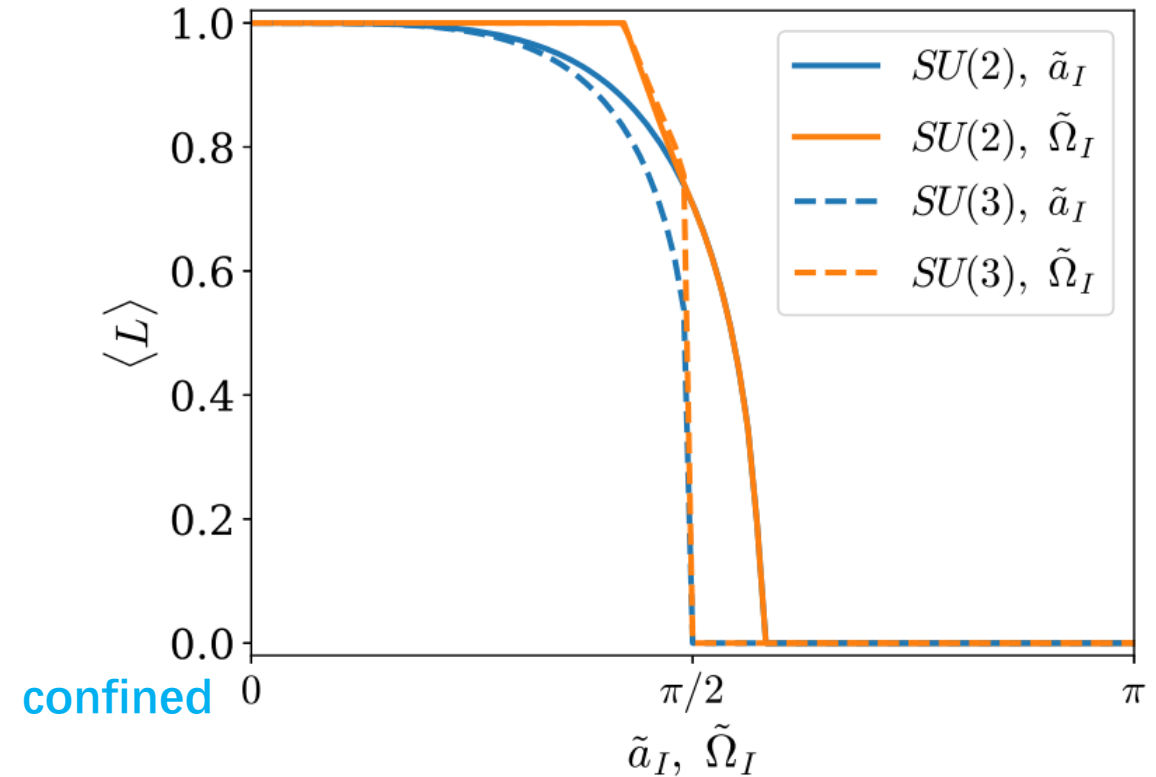
Acceleration:

$$\tilde{V}_{\text{eff}}(x) = 2T_{\text{loc}}^4 \left[\frac{2\pi^2}{3} \text{B}_4\left(\frac{\phi}{2\pi}\right) + \tilde{a}_I^2 \text{B}_2\left(\frac{\phi}{2\pi}\right) + \dots \right]$$

Rotation:

$$V_{\Omega}|_{\tilde{r}=0} = \frac{2\pi^2 T_0^4}{3} \sum_{s=\pm 1} \text{B}_4 \left[\left(\frac{\phi + s\tilde{\Omega}_I}{2\pi} \right)_{\text{mod } 1} \right]$$

deconfined



confined

No period for the acceleration case

Summary II

- ✓ The optical formulation yields the physical Polyakov-loop potential, consistent with the Rindler energy–momentum tensor through thermodynamic relations.
- ✓ Real acceleration strengthens the preference for the deconfined minimum
- ✓ Imaginary acceleration favors a center-symmetric phase, with similarities to and distinct differences from imaginary rotation.